Exam. Code : 211001
Subject Code : 4850

M.Sc. Mathematics 1st Semester

DIFFERENTIAL EQUATIONS

Paper—MATH-555

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions consisting **ONE** question from each section and **fifth** question can be attempted from any section.

SECTION—A

1. (i) Find the characteristic values and characteristic functions of the Strum-Liouville Problem

$$\frac{d}{dx}\left[x\frac{dy}{dx}\right]+\frac{\lambda}{x}y=0,\ y(1)=0,\ y(e^{\pi})=0. \quad 10$$

(ii) Find the necessary and sufficient condition for a second order differential equation to be self-adjoint and find the adjoint equation of the differential equation $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$. 10

2. (i) State and prove Strum Separation Theorem. 10

(ii) (a) Find the orthogonal trajectories of the family of parabolas $y - cx^3$ and draw its diagram. 5

(b) Compute the first four successive approximations of the problem :

$$y' = y^2, y(0) = 1.$$ 5

SECTION—B

3. (i) Solve the initial-value problem using Laplace transform :

$$\frac{d^2Y}{dt^2} - 2\frac{dY}{dt} - 8y = 0, \; Y(0) = 3, \; Y'(0) = 6.$$ 10

(ii) State and prove second shifting theorem and find L{G(t)} if :

$$G(t) = \begin{cases} 0, & 0 < t < \pi/2 \\ \sin t, & t > \pi/2. \end{cases}$$ 10

4. (i) State and prove Convolution theorem for Laplace Transforms. 10

(ii) (a) Find :

$$L^{-1}\left\{\frac{1}{s(s^2+1)}\right\}.$$ 5

(b) Find :

$$L\{e^{at} \sin^2 bt\}.$$ 5

SECTION—C

5. (i) (a) Define Fourier transform and its inverse transform. 2

(b) State and prove the change of scale property of Fourier transform. 4

(c) What is meant by self-reciprocal with respect to Fourier transform ? Also give an example of it. 4

(ii) State Parseval's identity for Fourier transforms.

Hence use it to evaluate $\int_0^\infty \frac{x^2}{(x^2+a^2)^2}dx$.

10

6. (i) Find the Fourier transform of

$$f(x)=\begin{cases} a^2-x^2, & |x|<a \\ 0, & |x|>a \end{cases}$$. Hence deduce that

$$\int_0^\infty \frac{\sin t - t\cos t}{t^3}dt=\frac{\pi}{4}.$$ 10

(ii) Solve the differential equation

$\frac{d^2y}{dt^2}-2\frac{dy}{dt}+y=\cos t$ using Fourier transform. 10

SECTION—D

7. (i) Find the generating function for Hermite polynomial and obtain recurrence relation connecting the Hermite polynomial of different degrees and their differential coefficients using generating functions. 10

(ii) (a) Express $f(x) = x^4 + 3x^3 + 4x^2 - x + 2$ in terms of Legendre Polynomials. 7

(b) Show that $P'_n(-1) = (-1)^{n+1} \frac{1}{2} n(n+1)$. 3

8. (i) State and prove Rodrigue's Formula for Laguerre Polynomials. 10

(ii) (a) Evaluate $J_0(1)$ and $J_0(4)$ upto 3 decimal places. 5

(b) Verify that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ satisfies Bessel equation of order $\frac{1}{2}$. 5